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LBC EXPOSURE TIME CALCULATOR

Program
This document deals with the formulas of the Exposure Time Calculator (ETC) for LBC.
Description
The ETC is organized in three panels: the main upper panel summarizes the input parameters, which should be necessary filled by the common user. The second (left) panel operates with the Total Exposure Time, the Signal to Noise ratio and the Magnitude of a given object. The third (right) panel operates with the Single Exposure Time, the number of exposures, the background and the magnitude of saturation.

1 First Panel: Input Parameters

The input parameters are: the filter, the seeing, the photometric aperture, the morphology of the source, the airmass and the moon phase. The filters available are divided into two main groups: the filters for the Blue Channel of LBC (U, Un, B, V) and those for the Red Channel (V, R, I, Gunn Z, Y). The ETC requires the seeing of the observation and the Photometric Aperture (PA) for the Photometry. The source type can be set to *star*, *elliptical galaxy* or *spiral galaxy* (the half light radius of the galactic sources is 0.4 arcsec). The Signal to Noise ratio is computed for a given PA. It is possible to select also the Airmass and the Moon Phase for the observation.

The second panel (left) deals with the computation of the Total Exposure Time (TET), the Signal to Noise Ratio (S/N) and the Magnitude (Mag_{tot}) for a given observation.
The third panel (right) deals with the computation of the Single Exposure Time (SET), the Number of exposures (N_{exp}), the Background and the Magnitude at which a given exposure is saturated (Mag_{sat}). The two panels are roughly independent.

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2 Second Panel: Total Exposure Time (TET)

For the TET panel, you need to fix two parameters to find the third one. These are the formulas linking the total exposure time, the Magnitude of a source and the Signal to Noise Ratio.

2.1 First Case: Magnitude at a given S/N and TET

First case: given the seeing, the aperture for photometry, the exposure time and the Signal to Noise ratio, you can compute the magnitude of a source in the given aperture and the total magnitude, given the aperture correction.
s=seeing (arcsec)

p=pixel size (arcsec/px)
t=total exposure time (second)
w=photometric aperture diameter (arcsec)
r=ratio between aperture w and seeing s (aperture in seeing units)
A=area (pixel)
M=total magnitude (AB mag)
Ma=magnitude at a given aperture (AB mag)
Ms=magnitude of the sky (AB mag/$arcsec^2$)
SN=Signal to Noise Ratio
F=total flux of the source in ADU
B=flux of the background (per pixel) in ADU
ZpAirm=magnitude zero point for a given Airmass
Zp=magnitude zero point for Airmass=0.0
Ca=correction aperture (for star, elliptical or spiral galaxy)
n=number of exposures
g=gain (e-/ADU)
a=flat field accuracy for a single exposure
ron=Read Out Noise of LBC (e-)

$p = 0.23$

$a = 0.005$

$ron = 12.0$

$g = 1.75$

$r = w/s$

$A = \pi(\frac{s*r}{2*p})^2$

$B = t * p^2 * 10^{-0.4(Ms-Zp)}$

$\eta = \frac{a^2}{n}$

$\alpha = 1$

$\beta = -(SN)^2/g$

$\gamma = -(SN)^2 \left[n * A * (ron/g)^2 + B * A/g + \eta * B^2 * A\right]$

$F = \frac{-\beta+\sqrt{\beta^2-4*\alpha*\gamma}}{2*\alpha}$

$Ma = -2.5 * \log_{10}(F) + ZpAirm$

$M = (Ma - Ca)$

Ma is the magnitude at a given SN in an aperture w, whereas M is the total magnitude of the object.

Description: given the seeing s, the pixel size p of LBC and the aperture in arcsec w, the ETC program computes the ratio r between the aperture and the seeing. Then it computes the area A in pixel for this aperture. The next step is to compute the background B in ADU for the single pixel, given the total exposure time t, the Magnitude of the Sky Ms, the Zero point Zp at $airmass = 0.0$. At the end one computes the magnitude limit Ma at a given Signal to Noise ratio SN. The magnitude of the sky Ms depends on the Moon day (0,3,7,10,14) and the filter used. It is always computed at airmass=0. It can be read from $SKYMAG.dat$ or from the LBC Database.

The Zero Point for t seconds of exposure time, for a given airmass and for a given filter is

Table 1: Magnitude of the Sky for given Moon Phase for different filters

Filter	$mag0$	$mag3$	$mag7$	$mag10$	$mag14$
LBC_U	22.895	22.429	20.860	19.453	17.987
LBC_Un	22.941	22.464	20.874	19.450	17.975
LBC_B	22.577	22.352	21.498	20.510	19.285
LBC_V	21.776	21.657	21.336	20.708	20.002
LBC_R	21.101	21.039	20.830	20.493	20.064
LBC_I	20.399	20.386	20.187	19.964	19.658
LBC_Z	19.315	19.313	19.189	19.056	18.872
LBC_Y	18.750	18.748	18.624	18.491	18.307

$$ZpAirm = 2.5 * \log_{10}(t) + ZpAirm(t = 1).$$

The zero point for 1 second of exposure time and for a given airmass $ZpAirm(t = 1)$ can be found in $ZEROPNT.dat$, where there are the zero points for airmass= 0.0, 1.0, 1.1, 1.2 and so on till 3.0; other values of airmass can be extrapolated.

The number of exposures n is computed dividing the Total Exposure Time TET by the Single Exposure Time SET for a single exposure. This parameter is taken from the right panel and it is the only parameter calculated by a cross talk of the two panels (TET and SET).

At the end one has the magnitude limit Ma for a given filter, moon, exposure time, airmass, seeing, aperture and Signal to Noise Ratio. These formulas compute the magnitude limit in an aperture w! If the object is greater than that aperture, the total magnitude of the object could be brighter. It is computed as a simple formula $Mag_{tot} = Ma - Ca$ and displayed in the outputs of the ETC for the TET panel.

2.2 Second Case: S/N for a given Magnitude and TET

Second case: given the seeing s, the aperture for photometry w, the total exposure time t and the total magnitude of an object M, you can compute the Signal to Noise Ratio SN. This result depends on the type of the source: star, elliptical galaxy or spiral.

s=seeing (arcsec)
p=pixel size (arcsec/px)
t=total exposure time (second)
w=photometric aperture diameter (arcsec)
r=ratio between aperture w and seeing s (aperture in seeing units)
A=area (pixel)
M=total magnitude (AB mag)
Ma=magnitude at a given aperture (AB mag)
Ms=magnitude of the sky (AB mag/$arcsec^2$)
SN=Signal to Noise Ratio
F=total flux of the source in ADU
B=flux of the background (per pixel) in ADU
ZpAirm=magnitude zero point for a given Airmass
Zp=magnitude zero point for Airmass=0.0
Ca=correction aperture (for star, elliptical or spiral galaxy)
n=number of exposures
g=gain (e-/ADU)
a=flat field accuracy for a single exposure
ron=Read Out Noise of LBC (e-)

$$p = 0.23$$

$$a = 0.005$$

$$ron = 12.0$$

$g = 1.75$

$r = w/s$

$A = \pi(\frac{s*r}{2*p})^2$

$B = t * p^2 * 10^{-0.4(Ms-Zp)}$

$\eta = \frac{a^2}{n}$

$ZpAirm = 2.5 * \log_{10}(t) + ZpAirm(t = 1)$

$Ma = M + Ca$

$F = t * 10^{-0.4*(Mc-ZpAirm1)}$

or equivalently

$F = 10^{-0.4*(Mc-ZpAirm)}$

$SN = \frac{F}{\sqrt{(F+B*A)/g+n*A*(ron/g)^2+\eta*B^2*A}}$

Description: given the seeing s and the aperture w, first compute the sky brightness B and the area A using the above formulas. Then compute r, the ratio of the aperture and seeing. For a stellar source, find in the file *totcorr_star.dat* the row corresponding to $r = w/s$; the second column gives Ca, the correction for a given aperture. For an elliptical galaxy, find in the file *totcorr_ell.dat* the row corresponding to $r = w/s$; for a spiral galaxy, the corresponding file is *totcorr_spi.dat*. Search in the column corresponding to the seeing the correction Ca, for a given aperture and a given seeing. Then correct the input magnitude for Ca, compute the flux F in ADU for the given source and compute the Signal to Noise ratio SN. These formulas compute the Signal to Noise ratio in an aperture w. The term η provides the contribution of the flat field accuracy to the Noise of a given exposure. It depends on the number of exposures ($\frac{a^2}{n}$).
Warning: This is the reference formula for the ETC: the first and the third cases of this panel (TET) are derived inverting this formula. To enhance the Signal to Noise Ratio it is possible to act on the total exposure time t and/or on the number of exposures n.

2.3 Third Case: TET needed to reach a given Magnitude at a given SNR

Third case: given the seeing s, the aperture for photometry w, the Signal to Noise ratio SN and the magnitude of an object M, you can compute the exposure time required to reach SN in the given aperture. This result depends on the type of the source: star, elliptical galaxy or spiral.

s=seeing (arcsec)
p=pixel size (arcsec/px)
t=total exposure time (second)
w=photometric aperture diameter (arcsec)
r=ratio between aperture w and seeing s (aperture in seeing units)
A=area (pixel)
M=total magnitude (AB mag)
Ma=magnitude at a given aperture (AB mag)
Ms=magnitude of the sky (AB mag/$arcsec^2$)
SN=Signal to Noise Ratio
F_1=flux of the source in ADU for 1 second of exposure time
B_1=flux of the background (per pixel) in ADU for 1 second of exposure time
ZpAirm1=magnitude zero point for a given Airmass at 1 second of exposure time
Zp1=magnitude zero point for Airmass=0.0 at 1 second of exposure time
Ca=correction aperture (for star, elliptical or spiral galaxy)
n=number of exposures
g=gain (e-/ADU)

a=flat field accuracy for a single exposure
ron=Read Out Noise of LBC (e-)

$p = 0.23$

$a = 0.005$

$ron = 12.0$

$g = 1.75$

$r = w/s$

$A = \pi(\frac{s*r}{2*p})^2$

$B_1 = p^2 * 10^{-0.4(Ms-Zp1)}$

$Ma = M + Ca$

$F_1 = 10^{-0.4*(Ma-ZpAirm1)}$

$K_1 = \frac{F_1+A*B_1}{g}$

$\alpha = F_1^2 - (a^2/n) * (SN)^2 * B_1^2 * A$

$\beta = -(SN)^2 * K_1$

$\gamma = -(SN)^2 * n * A * (ron/g)^2$

$t = \frac{-\beta+\sqrt{\beta^2-4*\alpha*\gamma}}{2*\alpha}$

Description: given the seeing s and the aperture w for photometry, compute the ratio r between w and s. Compute the area A of the aperture and the correction for a given aperture Ca. Then correct the input magnitude of the source for the Ca value, compute the flux of the source F_1 for an exposure time of 1 second and B_1 for the sky background. Then compute the total time t needed to reach a given Signal to Noise ratio SN in an aperture w.
Warning: The coefficient α must be a positive number. If it is negative or null, no solution can be reached. In this case it is useful to increase the Magnitude of the source M or the number of exposures n. In the same way, it is possible to decrease the given Signal to Noise to reach convergence in the calculations.

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3 Second Panel: Single Exposure Time

These formulas link the Single Exposure Time (SET), the number of exposures, the Background and the Magnitude Saturation (Magnitude of a star that saturates in a single exposure). Only one parameter is needed to obtain the others.

3.1 First Case: Single Exposure Time is given

Given the single exposure time SET, compute the number of exposures n, the background B and the magnitude at saturation $Msat$ for a single exposure.

s=seeing (arcsec)
p=pixel size (arcsec/px)
SET=single exposure time (second)
TET=total exposure time (second)
w=photometric aperture diameter (arcsec)
r=ratio between aperture w and seeing s (aperture in seeing units)
A=area (pixel)
Msat=magnitude at saturation (AB mag)
Ma=magnitude at a given aperture (AB mag)
Ms=magnitude of the sky (AB mag/$arcsec^2$)
SN=Signal to Noise Ratio
F=total flux of the source in ADU
B=flux of the background (per pixel) in ADU
ZpAirm1=magnitude zero point for a given Airmass at 1 second of exposure time
Zp1=magnitude zero point for Airmass=0.0 at 1 second of exposure time
Ca=correction aperture (for star, elliptical or spiral galaxy)
n=number of exposures
β=Moffat Profile Parameter
R_0=scale length of Moffat profile
I(R)=intensity for a Moffat profile at a given radius R
Io=peak intensity for the Moffat profile I(R=0)
TF=total flux of Moffat profile

The Moffat profile is defined as follows:

$\beta = 2.5$

$R_0 = \frac{s}{2*p}$

$I(R) = Io\left[1 + (2^{1/\beta} - 1) * (R/R_0)^2\right]^{-\beta}$

$TF = \int_0^\infty 2\pi * R * I(R) * dR$

defining $\alpha = 2^{1/\beta} - 1$, we have

$TF = Io * \pi * \frac{R_0^2}{\alpha*(\beta-1)}$.

$Io = \alpha\frac{\beta-1}{\pi*R_0^2}$ is the maximum of the Moffat profile for total flux $TF = 1$.
Given all these relations, it is simple to compute the required parameters.

$n = TET/SET$

$B = SET * p^2 * 10^{-0.4(Ms-Zp1)}$

$Io = 2^{16} - B$

$TF = Io * \pi * \frac{R_0^2}{\alpha*(\beta-1)}$

$F1 = \frac{TF}{SET}$

$Msat = -2.5 * \log_{10}(F1) + Zp1$.

Description: the number of exposures n is the ratio between the Total Exposure Time TET and the Single Exposure Time SET. It should be an integer number, so it is possible that the product $SET * n$ is substantially different from TET. In this case a simple *Warning* is given.
Given the seeing of the observation s, and assuming a Moffat profile, one computes the maximum of the profile Io for an object with total flux $TF = 1$. Given the Magnitude of the Sky Ms and the Single

Exposure Time SET, the Background B is derived. The magnitude at saturation $Msat$ is computed for 65536 (2^{16}) ADU (full well capacity of a single pixel for LBC). The maximum of an image is the sum of the background and the peak of the source Io, for an exposure time of SET. Given the total flux TF required to saturate the frame for a Single Exposure Time (SET), one can compute the flux for 1 second $F1$ and then derive the Magnitude of Saturation $Msat$. If the ADUs are greater than 65536 (2^{16}) or equal, the image is saturated and an *Error* message is given.
Warning: check that the Single Exposure Time (SET) is consistent with the Total Exposure Time (TET). It is not possible that SET is greater than TET. If TET is large, verify that the combination of TET and SET gives an adequate number of exposures n.

3.2 Second Case: Number of Exposures is given

Given the number of exposures n, compute the Single Exposure Time SET, the background B and the magnitude at saturation $Msat$ for a single exposure.

SET=single exposure time (second)
TET=total exposure time (second)
n=number of exposures

$SET = TET/n.$

Description: the Single Exposure Time SET is derived dividing the Total Exposure Time TET by the number of exposures n. Then to derive the Background B for a Single Exposure and the Magnitude of Saturation $Msat$ the formulas are the same of the First Case.
Warning: check that the number of exposures n is consistent with the Total Exposure Time (TET). n should be an integer number. If TET is large, verify that the combination of TET and n gives an adequate value for the Single Exposure Time SET.

3.3 Third Case: Background is given

Given the Background B for a single exposure, compute the Single Exposure Time SET, the number of exposures n and the magnitude at saturation $Msat$ for a single exposure.

s=seeing (arcsec)
p=pixel size (arcsec/px)
SET=single exposure time (second)
TET=total exposure time (second)
w=photometric aperture diameter (arcsec)
r=ratio between aperture w and seeing s (aperture in seeing units)
A=area (pixel)
Msat=magnitude at saturation (AB mag)
Ma=magnitude at a given aperture (AB mag)
Ms=magnitude of the sky (AB mag/$arcsec^2$)
SN=Signal to Noise Ratio
F=total flux of the source in ADU
B=flux of the background (per pixel) in ADU
ZpAirm1=magnitude zero point for a given Airmass at 1 second of exposure time
Zp1=magnitude zero point for Airmass=0.0 at 1 second of exposure time
Ca=correction aperture (for star, elliptical or spiral galaxy)
n=number of exposures
β=Moffat Profile Parameter
R_0=scale length of Moffat profile
I(R)=intensity for a Moffat profile at a given radius R
Io=peak intensity for a Moffat profile I(R=0)
TF=total flux of Moffat profile

$SET = \frac{B}{p^2 * 10^{-0.4(Ms-Zp1)}}$

$n = TET/SET$

$Io = 2^{16} - B$

$R_0 = \frac{s}{2*p}$

$TF = Io * \pi * \frac{R_0^2}{\alpha*(\beta-1)}$

$F1 = \frac{TF}{SET}$

$Msat = -2.5 * \log_{10}(F1) + Zp1.$

Description: given the Background B for a single exposure, it is possible to compute the Single Exposure Time SET knowing the background for 1 second of exposure time. The number of exposures n is the ratio of TET and SET. The peak Io of a source at the saturation level is derived using the background B. Given the seeing of the observation s, and assuming a Moffat profile, one computes from the maximum of the profile Io the total flux TF. The magnitude at saturation $Msat$ is computed for 65536 (2^{16}) ADU (full well capacity of a single pixel for LBC). Given the total flux TF required to saturate the frame for a Single Exposure Time (SET), one can compute the flux for 1 second $F1$ and then derive the Magnitude of Saturation $Msat$. If the ADUs are greater than 65536 (2^{16}) or equal, the image is saturated and an *Error* message is given.
Warning: it is not possible to enter a Background B greater or equal to 65536 (2^{16}) ADU. If TET is large, verify that the combination of TET and B gives an adequate number of exposures n.

3.4 Fourth Case: Magnitude at Saturation is given

Given the Magnitude at Saturation $Msat$ for a single exposure, compute the Single Exposure Time SET, the number of exposures n and the Background B for a single exposure.

s=seeing (arcsec)
p=pixel size (arcsec/px)
SET=single exposure time (second)
TET=total exposure time (second)
w=photometric aperture diameter (arcsec)
r=ratio between aperture w and seeing s (aperture in seeing units)
A=area (pixel)
Msat=magnitude at saturation (AB mag)
Ma=magnitude at a given aperture (AB mag)
Ms=magnitude of the sky (AB mag/$arcsec^2$)
SN=Signal to Noise Ratio
F=total flux of the source in ADU
B=flux of the background (per pixel) in ADU
ZpAirm1=magnitude zero point for a given Airmass at 1 second of exposure time
Zp1=magnitude zero point for Airmass=0.0 at 1 second of exposure time
Ca=correction aperture (for star, elliptical or spiral galaxy)
n=number of exposures
β=Moffat Profile Parameter
R_0=scale length of Moffat profile
I(R)=intensity for a Moffat profile at a given radius R
Io=peak intensity for a Moffat profile I(R=0)
B1=sky counts for 1 second of exposure time
F1=total flux for 1 second of exposure time
I1=peak intensity for 1 second of exposure time
TF=total flux of Moffat profile

$B1 = p^2 * 10^{-0.4(Ms-Zp1)}$

$F1 = 10^{-0.4*(Msat-ZpAirm1)}$

$R_0 = \frac{s}{2*p}$

$I1 = \frac{F1*\alpha*(\beta-1)}{\pi*R_0^2}$

$SET = \frac{2^{16}}{B1+I1}$

$B = SET * p^2 * 10^{-0.4(Ms-Zp1)}$

$n = TET/SET$

Description: given the Background $B1$ for 1 second of exposure and the flux at saturation $F1$ for 1 second of exposure, it is possible to compute the peak $I1$ of a source that saturates the CCD. The Single Exposure Time SET is derived from the flux of the background and the source for 1 second of exposure time. The Background B for a single exposure is derived using the Single Exposure Time SET and the Magnitude of the Sky Ms. Then the number of exposures n is derived referring to the Total Exposure Time TET. If the ADUs are greater than 65536 (2^{16}) or equal, the image is saturated. **Warning**: if TET is large, verify that the combination of TET and $Msat$ gives an adequate number of exposures n.

In case of problems with the ETC, contact the **LBC-Webmaster Stefano Gallozzi**
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